Price Movements in Speculative Markets: Trends or Random Walks*

by SIDNEY S. ALEXANDER, *Professor of Industrial Management*

There is a remarkable contradiction between the concepts of behavior of speculative prices held by professional stock market analysts on the one hand and by academic statisticians and economists on the other. The professional analysts operate in the belief that there exist certain trend generating facts, knowable today, that will guide a speculator to profit if only he can read them correctly. These facts are believed to generate trends rather than instantaneous jumps because most of those trading in speculative markets have imperfect knowledge of these facts, and the future trend of prices will result from a gradual spread of awareness of these facts throughout the market. Those who gain mastery of the critical information earlier than others will, accordingly, have an opportunity to profit from that early knowledge.

The two main schools of professional analysts, the "fundamentalists" and the "technicians," agree on this basic assumption. They differ only in the methods used to gain knowledge before others in the market. The fundamentalist seeks this early knowledge from study of the external factors that lie behind the price changes. In a commodity market he tries to estimate the future balance of supply and demand for that commodity. In the stock market he studies general business conditions and the profit prospects for various industries, and for the individual firms within those industries, with special attention to new developments.

The "technician" operates on the same basic assumption, that facts existing at one time will govern the prices at some future time, but he operates in a different manner. He leaves to others the study of the fundamental facts in the reliance that as those others act on their knowledge there will be a detectable effect on the price of the stock or commodity. The technician, accordingly, studies price movements of the immediate past for telltale indications of the movements of the immediate future.

Both schools of analysts thus assume the existence of trends which represent the gradual recognition by the market of emergent factual situations - trends which, if they exist, must depend for

* The data gathering and processing underlying this research were supported by the Sloan Research Fund of the School of Industrial Management, Massachusetts Institute of Technology.

their existence on a lagged response of the market prices to the underlying factors governing those prices. It might, at first blush, seem possible that the trends arise not from a lagged response of the market price to the fundamental circumstances, but rather from a trend in those underlying circumstances themselves. Thus, although a stock's price might at all times represent a given multiple of its earnings, its earnings might be subject to a long run trend. If, however, there are really trends in earnings, so that an increase in earnings this year implies a higher probability of an increase next year than do stable or declining earnings, the stock price right now should reflect these prospects by a higher price and by a higher ratio of price to current earnings. Consequently, if there is no lagged response there should be no trend in prices. By a trend in this connection we mean a positive serial correlation of successive changes or, more generally, a probability of future price change dependent on present price change.

The professional analysts would certainly not subscribe to the notion that the best picture of the future movements of prices can be gained by tossing a coin or a set of coins. Yet that is just what the academic students of speculative markets say is the best way. The academic students of speculative markets have come to deny the very existence of trends in speculative prices, claiming that where trends seem to be observable, they are merely interpretations, read in after the fact, of a process that really follows a random walk. A price can be said to follow a random walk if at any time the change to be expected canbe represented by the result of tossing a coin, not necessarily a 50-50 coin, however. In particular, a random walk would imply that the next move of the speculative price is independent of all past moves or events.

This probabilistic view of speculative prices is consistent with the theoretical bent of economists who like to talk about perfect markets. If one were to start out with the assumption that a stock or commodity speculation is a "fair game" with equal expectation of gain or loss or, more accurately, with an expectation of zero gain, one would be well on the way to picturing the behavior of speculative prices as a random walk. But in fact, this picture of a speculative price movement is as much based on empirical findings as on theoretical predispositions. In a pioneer work Bachelier, [1] a student of the great French mathematician Poincaré, derived, in his doctoral thesis in 1900, a theory that speculative prices follow random walks, largely from the assumption of zero expectation of gain. He then compared the statistical distribution of price behavior expected from this theory with observed distributions of price changes of certain government securities (*rentes*) on the Paris *Bourse*, and he found a close correspondence between the observed distribution and that to be expected from his theory.

---

[1] M. L. Bachelier, Théorie de la Speculation, Gauthier-Villars, Paris, 1900.

The most impressive recent findings confirming the random walk hypothesis are those of Kendall.[2] He calculated the first twenty-nine lagged serial correlations of the first differences of twenty-two time series representing speculative prices. Nineteen of these were indexes of British industrial share prices on a weekly basis. (See Table 1). Two of the remaining three were cash wheat at Chicago, one weekly and one monthly, and the last was the spot cotton price at New York, monthly. Essentially, Kendall was asking with respect to each weekly series: How good is the best estimate we can make of next week's price change if we know this week's change and the changes of the past twenty-nine weeks and correspondingly for the monthly series?

Contrary to the general impression among traders and analysts that stock and commodity prices follow trends, Kendall found, with two or three exceptions, that knowledge of past price changes yields substantially no information about future price changes. More specifically, he found that each period's price change was not significantly correlated with the preceding period's price change nor with the price change of any earlier period, at least as far as he tested, up to twenty-nine periods. Essentially, the estimate of the next period's price change could have been drawn at random from a specified distribution with results as satisfactory as the best formula that could be fitted to past data. In the case of weekly wheat prices, that distribution was studied in detail and it turned out to be very close to a normal distribution.

There was one notable exception, however, to this pattern of random behavior of price changes. That was the monthly series on cotton prices in the United States since 1816 with, of course, a few interruptions for such events as the Civil War. For this series there did appear to be some predictability, and Kendall felt impelled to draw the moral that it is dangerous to generalize even from fairly extensive sets of data. For, from the behavior of wheat prices and the stock prices, one might have concluded that speculative markets do not generate autocorrelated price changes - and here was cotton providing a notable exception.

Alas, Kendall drew the wrong moral. The appropriate one is that if you find a single exception, look for an error. An error there was, for the cotton price series was different from the others investigated by Kendall. Almost all the others were series of observations of the price at a specified time - say, the closing price on Friday of each week. Each observation of the cotton series was an average of four or five weekly observations of the corresponding month. It turns out that even if the original data - the Friday closing prices - were a random walk, with successive first differences uncorrelated, the first differences of the monthly average of

[2] M. G. Kendall, "The Analysis of Economic Time Series - Part I: Prices." Journal of the Royal Statistical Society (Series A), Vol. 96 (1933), pp. 11-25.

TABLE 1

SERIAL CORRELATIONS OF FIRST DIFFERENCES OF KENDALL'S STOCK PRICE INDEXES, 1928-1938, FOR VARIOUS INTERVALS OF DIFFERENCING

Kendall's No.	Industry Group	First Order Serial Correlations. Differences Taken Over Interval of:					Second Order Serial Correlations
		1 Week	2 Weeks	4 Weeks	8 Weeks	16 Weeks	8 Weeks
1	Banks and Discount Companies	0.058	0.092	-0.067	-0.059	0.191	0.136
2	Insurance Companies	.052	.048	- .056	- .169	- .035	.049
3	Investment Trusts	.301	.450	.279	.020	.066	.064
4	Building Materials	.125	.044	- .087	- .075	.345	.271
5	Coal	.148	.156	- .014	- .186	- .074	.032
6	Cotton	.087	.179	.036	- .227	- .075	.123
7	Electric Light and Power	.181	.145	- .024	- .130	.219	.232
8	Gas	.096	.235	.103	.119	.360	.232
9	Iron and Steel	.088	- .005	- .028	- .122	- .009	.057
10	Oil	- .013	.027	.015	- .015	.073	.061
11	Total Industrial Productive	.195	.155	- .016	- .149	.055	.110
12	Home Rails	.010	- .030	- .002	.162	.072	- .019
13	Shipping	.053	.032	- .017	.111	.388	.325
14	Stores and Catering	.230	.145	- .082	- .218	.104	.199
15	Total Industrial Distributive	.237	.179	- .008	.033	.303	.241
16	Breweries and Distilleries	.034	.102	.046	.020	.152	.078
17	Miscellaneous	.200	.191	- .011	- .101	.267	.267
18	Total Industrial Miscellaneous	.177	.190	.033	- .069	.291	.252
19	Industrials (All Classes Combined)	.234	.207	.018	- .063	.279	.246

four or five of these weekly observations would exhibit first-order serial correlations of about the magnitude Kendall found for cotton.[3] So Kendall's exception vanishes, and we are left with the conclusion that at least for the series he investigated the serial correlations were not significantly different from zero.[4]

But the question immediately arises whether a week is not an inappropriate period of observation. The market analysts might protest that when they speak of a trend they are speaking of a smooth underlying movement on which is typically superimposed a lot of short-term fluctuation. With weekly observations the short-term fluctuations might very easily swamp the underlying trends. In particular, the give and take of the market leads to a phenomenon, recognized by all analysts, of reactions, usually called technical reactions, presumably associated with profit taking. These reactions are, of course, negatively correlated with the main price swings. That's what makes them reactions. Kendall's correlations, close to zero, could possibly be a consequence of the combination of the negative contributions of the reactions and the positive contributions of the trends.

The path of a speculative price might, accordingly, be represented by a sum of two components, a smooth underlying trend or cycle changing direction only infrequently, and a much shorter cycle of action and reaction. Under this hypothesis the first-order serial correlations of daily price changes might be negative, the first order correlation of weekly changes might be close to zero, while the first order serial correlations of monthly or bimonthly changes might be significantly larger than zero.

We can test this possibility by studying the first order serial correlations of Kendall's data using successively longer intervals of differencing. As we do so, and consider first the first order serial correlation of one week changes, then of two week, four week, eight week, and sixteen week changes, the influence of the reactions should become smaller and smaller and the trend effect, if there is one, should become dominant. The corresponding correlations, roughly calculated,[5] are given in Table 1.

---

[3] This point was independently discovered by the author and by Holbrook Working. The latter, however, had the pleasure of first publishing it in "Note on the Correlation of First Differences of Averages in a Random Chain," Econometrica, Vol. 28, No. 4, October 1960, pp. 916-918.

[4] Another possible exception may be noted for Kendall's Series 3, Investment Trusts, whose first five serial correlations were 0.301, 0.356, 0.158, 0.164 and 0.066. This series will be mentioned again below.

[5] Roughly, because they were computed, not from the original data, but from the serial correlations published by Kendall. Since successive serial correlations are based on fewer observations because of the necessity of sacrificing end terms, a certain "end term error" is introduced by this procedure.

While an occasional high value correlation occurs in Table 1 for intervals greater than one week, it must be remembered that for the given total time period under study, the number of observations drops in proportion to the length of the differencing interval. Since the variance of the correlation coefficient is inversely proportional to the number of observations, it is directly proportional to the length of the differencing interval. An occasional high computed value of the correlation becomes increasingly probable as the differencing period is lengthened, even if the true correlation is zero.

It must be concluded that the data of Table 1 do not give any substantial support to the hypothesis that, as differences are taken over longer and longer intervals, the first order serial correlations of the first differences generally increase.

Once again, Series 3, investment trusts, is an exception. It seems to have a particularly high level of serial correlation on a two-week period of differencing. Other occasional higher values among the various series, for changes at sixteen week intervals in particular, have to be seriously discounted although they suggest intriguing possibilities for further study.

These higher correlations for sixteen week changes (for Series 4, 8, 13, 15, and possibly 17, 18, and 19) proceed from a rather curious relationship that holds for 18 of the 19 series. The first order correlations on an eight-week basis tend to be algebraically smaller than the second order correlations (see Table 1). The implication of the existence of an eight week half cycle may be an interesting suggestion, although it could hardly be said to be established by the data.

One further attempt was made, in spite of Kendall's findings that the serial correlations were not significantly different from zero, to see if some nugget of systematic trend behavior might still be found in his data. It is possible that while the lagged autocorrelations of any series were not found to be significantly different from zero when taken one at a time, they might jointly form a pattern that is significant. A simple test of this possibility was attempted. A trend was fitted to the first differences of each stock price series by a Spencer 21-term moving average.[6] Then the ratio of the variance of the moving average to the variance of the first differences themselves was computed.

The variance ratios given in Table 2 are to be interpreted as follows. If each first difference lay exactly on the moving average

---

[6] See E. T. Whitaker and G. Robinson, The Calculus of Observations, (4th ed.) London 1944, p. 290, for the formula used. Actually, it was not necessary to fit the trends to the series themselves, but the variance of the moving average, expressed in units of the variance of the first differences, could be computed by applying the smoothing coefficients of the Spencer formula directly to the lagged serial correlations.

TABLE 2

RATIOS OF THE VARIANCE OF SMOOTHED FIRST DIFFERENCES TO THE VARIANCE OF UNSMOOTHED FIRST DIFFERENCES KENDALL'S STOCK PRICE INDEXES

(Smoothing Performed by 21 Term Spencer Moving Average)

Series	Industry Group	Ratios
1	Banks and Discount Companies	0.161
2	Insurance Companies	.158
3	Investment Trusts	.525
4	Building Materials	.162
5	Coal	.202
6	Cotton	.210
7	Electric Light and Power	.206
8	Gas	.230
9	Iron and Steel	.158
10	Oil	.145
11	Total Industrial Productive	.212
12	Home Rails	.138
13	Shipping	.151
14	Stores and Catering	.212
15	Total Industrial Distributive	.219
16	Breweries and Distilleries	.171
17	Miscellaneous	.221
18	Total Industrial Miscellaneous	.218
19	Industrials (All Classes Combined)	.232

trend line, that is, if the trend line were a perfect fit, the variance ratio would be unity. If, on the other hand, all the serial correlations of order greater than zero were identically zero, the expected values for a random walk, the variance ratio would be 0.143, the sum of the squares of the coefficients in the smoothing formula. It is, of course, possible for the ratio to be even less than 0.143.

Except for Series 3, the trend variance is not a much larger proportion of the original variance of the first differences than would be expected in the case of a random walk. It must be concluded that, with this exception, if trends exist in the first differences, they are very weak.

All in all, Kendall's data do seem to confirm the random walk hypothesis. Further work by Osborne[7] strengthens the random walk hypothesis from a different point of view. While Kendall

[7] M. F. M. Osborne, "Brownian Motion in the Stock Market," Operations Research, Vol. 7, No. 2, March-April 1959, pp. 145-173. See also comment and reply in Operations Research, Vol. 7, No. 6, November-December 1959, pp. 806-811.

worked with serial correlations for each series separately, Osborne worked with ensembles of price changes. Roughly stated, he found that the changes in the logarithms of stock prices over any period in a given market, principally the New York Stock Exchange, constituted an ensemble which appeared to be approximately normally distributed with a standard deviation proportional to the square root of the length of the period. This proportionality of the standard deviation of price differences to the square root of the differencing period is a characteristic of a random walk and had been pointed out much earlier by Bachelier. In Bachelier's case, however, the differences were arithmetic, while in Osborne's they were logarithmic.

It must be noted that Osborne's measurements do not concern trends in the prices of stocks but merely the statistical distribution of the changes in the logarithms, which, as Osborne pointed out, correspond quite closely to percentage changes. That they do not correspond exactly to percentage changes has an important bearing on one of Osborne's principal findings, as we shall see.

Osborne also supplied a theoretical mechanism that could explain the observed pattern of price movements. The mechanism is a random walk in the logarithm of prices with each step being a constant logarithmic value, depending on the time length of the step. The basic step is a transaction of which there might be ten or a hundred a day. The compounding of such steps in familiar probability sequences would, over any period of time, yield a normal distribution of changes in the logarithms of price, with standard deviation proportional to the square root of the period over which comparison is made.

One peculiar result of Osborne's proposed mechanism merits further study. Bachelier, the pioneer in regarding speculative price behavior as a random walk, derived the theoretical properties of the distribution of changes in the prices of *rentes* on the assumption of a "fair game," that is a zero expectation of gain. A price change in either direction of a given amount was equally probable in Bachelier's model. Osborne made a somewhat different assumption with a radically different result; he assumed that a change in either direction of a given amount in the logarithm of price was equally likely, no longer a fair game.

Thus, under Bachelier's assumption, given an initial investment value, say \$100, it would be equally probable, at the end of time T to be worth \$100 + k or \$100 − k. Exactly how large k would be for any stated probability would depend on the fundamental constant of the distribution and the square root of the length of time T. But whatever the value of k, so long as the probability of a gain of k is equal to the probability of a loss of k for all k within the permitted range, the expected value in any future period remains \$100 and the expected gain 0.

Under Osborne's assumption, however, there will be an expectation of gain. Suppose, using logarithms to base 10 and starting from $100, a gain over some particular period of time, say five years, is equally likely to be a gain of logarithm 1 or loss of logarithm −1. These would correspond to an equal probability that the price at the end of five years would be either $10 or $1,000. The mathematical expectation in this case would be $505, or an expected gain of $405. This example illustrates the familiar difference between the arithmetic and the geometric mean. Over the long run, then, it makes a great deal of difference whether there is an expectation of zero arithmetic gain or zero logarithmic gain. In the latter case there will be a tendency for an investment value to grow, independent of any growth in the economy other than the growth implicit in the existence of a random walk in the logarithms.

How clearly established an empirical finding is the logarithmic rather than the arithmetic step in the random walk? Osborne was led to the logarithmic form, while Bachelier was not, because Bachelier considered only a single type of security at a time, whereas Osborne considered an ensemble of prices, usually all the common stock prices on a particular exchange. Osborne assumed, without much explicit consideration, that it was appropriate to try to fit one distribution of expected change to all common stocks, whether priced at $100 or $10 or whatever. Very little empirical investigation is required to show that the relative frequency of price rises of $10 in one month is much smaller among stocks selling at $10 than among stocks selling at $100. On the other hand, it is quite reasonable to expect that the relative frequency of a $1 price rise in a month among $10 stocks would be about equal to the relative frequency of a $10 price rise in a month among $100 stocks. And rough empirical tests seem to be in accord with the latter case.

If then we have to choose a single distribution that will fit stocks of all different prices, and if our only choice were between equal probabilities of dollar amount changes and equal probabilities of proportional changes, we are necessarily led to choose the latter. The assumption of equal probabilities of given changes in the logarithm of price falls in the latter class.

But there are other possible models which yield equal probabilities of changes of given proportions. One is of particular interest to us in that it certainly fits the data as well as the logarithmic model and does not imply a built-in growth of values as does the logarithmic. It postulates equal probability of given percentage changes, almost the same as equal probability of given logarithmic changes, but not quite. On the tiny difference hinges the existence or nonexistence of the remarkable property of speculation being a game biased in favor of winning.

In both the logarithmic form and the percentage form of the hypothesis it is equally probable that a $100 stock rises by $10 in a month or that a $10 stock rises by $1. Under both schemes it is equally probable that a $100 stock declines by $10 in a month or a $10 stock by $1. But under the percentage form it is equally probable that a $100 stock goes to $101 or to $99 in a given time, whereas in the logarithmic form it is equally probable that a $100 stock goes to $101 or $99.01 in a given time. This difference of one cent in a dollar change from $100 spells the difference between zero expectation of gain and positive expectation.

If, then, the percentage hypothesis is adopted instead of the logarithmic hypothesis, the expectation of gain disappears. The difference between the distributions generated by the two hypotheses would, over most time periods of practical interest, be so fine that any test delicate enough to distinguish between them is likely to throw them both out.

In fact, both hypotheses would generate normal distributions of the changes in the logarithms of prices, differing only in their means and standard deviations. In testing various models of this sort we generally infer the mean and the standard deviation from the data and assume that the mean was influenced by general economic conditions separate from the random walk. Under these circumstances we can say whether the observed distribution is or is not close to normal, but we cannot say whether it is closer to the percentage hypothesis or to the logarithmic. To discriminate between these hypotheses we would need an independent measure of the random step, or of the standard deviation. Bachelier actually derived such an independent measure from the price of options, but Osborne merely showed that the distribution resembled a normal distribution and the standard deviation increased with the square root of the differencing period. To whatever extent his findings support the logarithmic hypothesis, they also support the percentage hypothesis.

But Osborne did not rigorously test the normality of the distribution. A rigorous test, for example the application of the chi-square test to some of the data used by Osborne, would lead us strongly to dismiss the hypothesis of normality. (See Table 3). It yields a chi-square of over 60 for 8 degrees of freedom, although almost all of the discrepancy between actual and expected frequencies arises from the extreme classes of increases or decreases greater than 10 per cent.[8] It may be presumed that special factors operated to produce far more large price changes than are characteristic of a normal distribution. This sort of situation (leptokurtosis) is frequently encountered in economic statistics and would certainly overshadow any attempt to test fine points such as the difference between a logarithmic and a percentage scheme.

---

[8] It should be noted that Osborne remarked that the tails of the observed distribution did not appear to correspond to those of the normal distribution.

TABLE 3

CHANGES IN PRICES OF LISTED COMMON STOCKS (NYSE) OBSERVED COMPARED WITH EXPECTED

(Month Ending November 15, 1956)

Percentage Price Change	No. of Issues	
	Observed [a]	Expected [b]
+ 10% and over	54	21
+ 8% to + 10%	30	30
+ 6% to + 8%	50	54
+ 4% to + 6%	71	87
+ 2% to + 4%	119	127
− 2% to + 2%	346	328
− 4% to − 2%	149	157
− 6% to − 4%	100	121
− 8% to − 6%	74	79
− 10% to − 8%	31	43
− 10% or over	51	28
TOTAL	1075	1075

[a] Source: The Exchange (NYSE) Dec. 1956, back cover.

[b] On the assumption of normal distribution with mean equal to the median of the observed distribution, with standard deviation equal to the semi-interquartile range divided by 0.6745, and with total frequency equal to observed total frequency.

In any case the requirement that equal proportional gains for stocks of different prices should have equal probability does not imply a nonzero expectation. The nonzero expectation follows specifically from the assumption of unequal steps, measured in dollars, in the random walk, as between steps up and steps down. For small steps, equal logarithmic changes imply almost equal percentage changes, but the very small difference eventually grows to a very large one, another demonstration of the wonderful power of compound interest.

But more to the point than the difference between logarithmic and percentage schemes is the question: How far do Osborne's findings go to show that stock market prices really follow a random walk? Osborne, at best, merely showed that stock price changes might, to a rough approximation, have been generated by a random walk type of model. He suggests conditions that are sufficient to generate the observations, but not necessary. In plain language, he has a scheme which could have generated his observations; maybe it did and maybe it didn't.

It does seem that the principal clash between the analyst and the academic concerns not the nature of the statistical ensemble of price changes but rather the question of the existence of trends. For, if the implication of Kendall's findings are indeed general, it would make as much sense to try to predict the outcome of a coin toss as to predict movements of the stock market. The prime issue is, therefore, whether there is some way in which speculative price behavior is not random.

In order to attack this problem, the author ran off a set of simple tests of randomness of successive monthly or weekly changes of speculative prices. The technique may be illustrated by a weekly series on wheat (Wednesday closing prices of cash wheat over the period 1883-1934, excluding 1915-1920, a total of 2,379 weeks). Each week was classified as being a week of price rise or price fall, and the lengths of the runs were tallied. A run is defined as a sequence of successive weeks in which the price moves in the same direction. Table 4 shows the resulting table of runs and compares it with the distribution to be expected on the assumption of a random walk with equal probability of rise or fall.

TABLE 4

DISTRIBUTION OF LENGTHS OF RUN OF WEEKLY CASH WHEAT PRICES AT CHICAGO [(a)]

(1883-1934, Excluding 1915-1920)

Length of Run (Weeks)	Observed		Expected [(b)] Up or Down
	Up	Down	
1	280	295	297
2	147	132	149
3	86	77	74
4	38	42	37
5	15	18	19
6	13	12	9
7 or longer	7	8	9
TOTAL	586	584	595 [(c)]

[(a)] Source: Holbrook Working, "Prices of Cash Wheat and Futures at Chicago Since 1883," Wheat Studies Vol. II, No. 3, November 1934, pp. 75-124. See also Kendall, op. cit., Table 1 for frequency distribution for these prices, of differences between week t and t + 1 against differences between weeks t + 1 and t + 2.

[(b)] Expected on assumption of 0.5 probability of rise or fall, and 2,379 monthly observations.

[(c)] Expected total differs from sum because of rounding.

The correspondence is very close indeed, suggesting that at least the sequence of directions of changes in weekly wheat prices might have been produced by a random walk. Kendall[9] previously showed that, except for a few extreme items, the distribution of the size of these weekly wheat price changes is normal and independent of the movement of the preceding week.

The distribution of the runs of Standard and Poor's monthly composite index of stock prices,[10] 1918-56, is, however, inconsistent with the assumption of a random walk of equal probability of rise or fall. (Expectation 1 for 1918-56 in Table 5). We may note, however, that for this stock average for the period 1918-56:

(a) The relative frequencies of rising and declining months were 0.58 and 0.42, respectively;

(b) The relative frequency of rising months among all months for which the preceding month was rising, $p(+|+)$, was 0.67, and the relative frequency of declining months for which the preceding month was declining, $p(-|-)$, was 0.50.

A new set of expected runs based on these contingent relative frequencies for 1918-56 did fit the observed runs quite well. (Expectation 2 for 1918-56 in Table 5). Furthermore, expectations based on these relative frequencies derived from the 1918-56 data fit the 1871-1917 data for upruns quite closely, but downruns only fairly well. (Expectation 2 for 1871-1917 in Table 5). Actually, the simpler hypothesis (Expectation 3) that the probability of continuation of a run is 0.6 and of termination 0.4 fits the 1871-1917 data very well and also the 1918-56 upruns. It does imply a much higher frequency of long downruns than was observed in 1918-56, however.

Unfortunately, this evidence of the probability of one month's movement depending on the previous month's is entirely the result of using an average of weekly prices for each month's observation. Distributions of runs of industrial stock prices for 1897-1929 and 1929-59 are given in Table 6, based on a single-point observation for each month, the last trading day of the month. These distributions are very close to those to be expected on the assumption of each month's change being independent of the previous month's, with a 0.57 probability of a monthly rise and 0.43 of a decline. (Expectation 4, Table 6). The same probabilities seem to fit both periods equally well, and the hypothesis of trends seems to be blown sky high.

One notable exception to the rule that statisticians have not found trends in stock market prices is furnished by the work of

---

[9]Op. cit.

[10]The index is a monthly average of weekly observations.

TABLE 5

LENGTHS OF RUN OF STANDARD & POOR'S MONTHLY COMPOSITE STOCK PRICE INDEX,[a] OBSERVED (Obs) vs. EXPECTED (Ex)

(Monthly Averages of Weekly Indexes)

Length of Run	January 1918 to March 1956								January 1871 to December 1917					
	Up				Down				Up			Down		
(Months)	Obs	Ex 1[b]	Ex 2[c]	Ex 3[d]	Obs	Ex 1[b]	Ex 2[c]	Ex 3[d]	Obs	Ex 2[c]	Ex 3[d]	Obs	Ex 2[c]	Ex 3[d]
	(Number of Runs)													
1	31	58	30	37	39	58	45	37	41	37	45	40	56	45
2	21	29	20	22	27	29	23	22	29	25	27	30	28	27
3	16	15	13	13	15	15	11	13	15	17	16	17	14	16
4	10	7	9	8	9	7	6	8	11	11	10	9	7	10
5	5	4	6	5	1	4	3	5	4	7	6	4	4	6
6 or longer	9	4	12	8	1	4	3	8	9	15	8	10	4	8
TOTAL	92	117	90	93	92	117	90[e]	93	109	112	112	110	112[e]	112

[a]Source: 1918-1956. Standard & Poor's Trade & Securities Statistics (Security Price Index Record). Number of Monthly observations: 467 for 1918-56, and 559 for 1871-1917.

[b]Based on probability of 0.5 for rise or fall, successive months independent, derived from *a priori* hypothesis.

[c]Based on $p(+|+) = 0.67$ and $p(-|-) = 0.50$, derived from 1918-56 relative frequencies.

[d]Based on $p(+|+) = p(-|-) = 0.6$, derived from 1871-1917 relative frequencies.

[e]Expected total, differs from sum because of rounding.

TABLE 6

DISTRIBUTION OF LENGTHS OF RUN OF MONTHLY INDUSTRIAL STOCK PRICES,[a] OBSERVED (Obs) vs. EXPECTED (Ex)

(End of Month Closing Prices)

Length of Run (Months)	Jan. 1897 - Jan. 1929				Feb. 1929 - Dec. 1959			
	Up		Down		Up		Down	
	Obs	Ex 4[b]	Obs	Ex 4[b]	Obs	Ex 4[b]	Obs	Ex 4[b]
	(Number of Runs)							
1	42	40	58	53	38	38	49	50
2	24	23	18	23	22	22	19	22
3	16	13	14	10	9	12	10	9
4	6	7	4	4	7	7	4	4
5	1	4	1	2	3	4	2	2
6 or longer	7	6	1	1	8	6	2	1
TOTAL	96	93	96	93	87	88[c]	86	88

[a]Dow Jones Industrials, Jan. 1897 to Jan. 1929 with break from July 1914 to Jan. 1915 (379 months). Source: The Dow Jones Averages, Barron's, New York 1931; Standard & Poor's Industrials, Feb. 1929 - Dec. 1959 (362 months). Source: see fn. (a), Table 5.

[b]Based on assumption of independence of successive monthly changes, and on the probability of 0.57 of a rise, and 0.43 of a fall, derived from entire period 1897 - 1959.

[c]Expected total differs from sum because of rounding.

Cowles and Jones.[11] They found an excess of sequences over reversals in stock market prices, but they used, at least for monthly data, averages of weekly observations. Their findings presumably derive principally from this error, plus a second effect, the influence of more frequent movements up than down.

The first effect is illustrated by the data in Table 5, based on monthly averages of weekly observations. For these data there is a ratio of 1.56 of sequences to reversals for 1871-1917, and 1.54 for 1918-56. The ratio of sequences to reversals shrinks to 1.045 for the 1897-1959 end of month data of Table 6. The second effect can largely be explained on the basis of a 0.57 relative frequency of monthly rises and 0.43 of monthly falls for 1897-1959. For if each monthly movement were drawn independently at random with these probabilities of rise and fall, we should expect a ratio of

[11]Alfred Cowles and Herbert E. Jones, "Some a Posteriori Probabilities in Stock Market Action," Econometrica, Vol. 5, No. 280, July 1937, pp. 280-294.

sequences to alternations of 1.040, almost identical to that observed.

In a revision of his findings designed to eliminate the first effect, Cowles still found some evidence, though weaker, of the existence of an excess of sequences over reversals, a ratio of 1.07 for monthly one point observations as compared with an earlier figure of 1.33 for the monthly averages of weekly observations.[12] But he did not make any allowance for the effect of a higher relative frequency of rises than declines. If the relative frequency of monthly rises were between 0.59 and 0.60 in the period covered by Cowles' data, as is likely, the ratio of 1.07 would be expected even if monthly movements were independent.

It must be concluded that the month to month movement of stock prices, at least in direction, is consistent with the hypothesis of a random walk with about a 6 to 4 probability of a rise. Evidence to the contrary was spurious, arising from the correlations introduced by monthly averaging or neglect of the unequal probability of rise and fall.

Lest the reader take undue comfort from the bullish implication of the 6 to 4 probability of a rise, it must be noted that declines, though less frequent than rises, are sharper, so that the average decline is substantially larger than the average rise. Nevertheless, there has been a well recognized upward trend of stock market prices of the order of 5.6% a year over the period 1897-1959. (Table 7).

One final test may be reported that should give great comfort to the analyst and encouragement to those who would use statistical studies to guide their speculative efforts, for it furnishes evidence that stock price changes could not have been generated by a random walk. Suppose we tentatively assume the existence of trends in stock market prices but believe them to be masked by the jiggling of the market. We might filter out all movements smaller than a specified size and examine the remaining movements. The most vivid way to illustrate the operation of the filter is to translate it into a rule of speculative market action. Thus, corresponding to a 5% filter we might have the rule: if the market moves up 5% go long and stay long until it moves down 5% at which time sell and go short until it again moves up 5%. Ignore moves of less than 5%. The more stringent the filter, the fewer losses are made, but also the smaller the gain from any move that exceeds the filter size. Thus with a 5% filter there will be a loss on any move between 5% and 10.53% and a gain on any move larger than 10.53%. For if the move is just a 10.53% move, say from 100 to 110.53, then we would

---

12 Alfred Cowles, "A Revision of Previous Conclusions Regarding Stock Price Behavior," Econometrica, Vol. 28, No. 4, October 1960, pp. 909-915. The data cover about 1,000 months over the periods 1834-1865, 1897-1922, 1928-1958.

go long at 105 (100 plus 5%) and sell at 105 (110.53 minus 5%) and so just break even. With a 10% filter most of the moves which entailed a loss with the 5% filter would be filtered out. But a 20% move, which would yield a 9% profit with a 5% filter (computed on the lower vertex of the move, actually about 8.6% of the purchase price), would yield a 2% loss on a 10% filter.

Thus, as the filter size is increased, the number of transactions is reduced, and losses on small moves are eliminated, gains on large moves are reduced, and some moves which would yield gains with a small filter will yield losses with a large. This example illustrates the familiar tradeoff between reliability of the information and the cost of the information. The more stringent the filter, the higher the reliability, but the more of the move that is sacrificed in identifying it both in getting in and in getting out.

The results of the application of various filters to the Dow Jones and Standard & Poor's industrial averages from 1897 through 1959 are shown in Table 7. If stock price movements were generated by a trendless random walk, these filters could be expected to yield zero profits, or to vary from zero profits, both positively and negatively, in a random manner. Given an underlying long term trend, they might be expected to produce some profits, the greater profits being associated with the greater filter, but in any case, profits smaller than could be expected from just buying and holding.

In fact, medium filters uniformly yield profits, and the smallest filters yield the highest profits, and very high they are.

The retrospective gains from the filter rule (before commission) are compared in Table 7 with the gains that could be achieved over the corresponding period by just buying and holding. The results uniformly favor the smaller filters over the buy and hold method. Thus, the filter method derives its success from a characteristic of stock price behavior other than that implied by the upward long term trend alone. This conclusion is also confirmed by the fact, apparent from inspection of the work sheets, that the filter method made gains on the declines as well as on the rises.

From a practical standpoint these profits would be substantially reduced, but by no means eliminated, by the payment of commissions. I leave to the interested reader the computation of allowance for commissions.

It must be concluded that there <u>are</u> trends in stock market prices, once the "move" is taken as the unit under study rather than the week or the month. That is, the nonrandom nature of stock price movements revealed by Table 7 proceeds not only from filtering out small moves, but also from transforming the measure over which changes are considered. The many statistical studies which have found speculative prices to resemble a random walk have dealt with changes over uniform periods of time. The filter operation, however, deals with changes of given magnitude irrespective of the

TABLE 7

PROFITS FROM FILTERS OF VARIOUS SIZES COMPARED WITH BUY AND HOLD [a]

(1897 - 1959) [b]

Period	Filter Size [c]											Buy & Hold
	5%	6%	8%	10%	12.5%	15%	20%	25%	30%	40%	50%	
	Average Move (%) [d]											
1897 - 1914	13.8	15.8	19.8	22.8	30.7	39.6	62.6	62.6	82.5	80.2	97.0	-
1914 - 1929	12.8	14.9	19.7	25.4	33.3	43.0	69.4	115.8	115.8	115.8	115.8	-
1929 - 1959	14.5	16.4	22.3	26.3	31.6	36.1	52.9	72.3	188.9	199.0	291.0	-
	Average Profit Per Transaction (%) [e] (Before Commissions)											
1897 - 1914	2.9	3.0	2.7	1.5	3.2	5.4	12.2	4.0	7.7	(9.2)	(15.5)	75.3
1914 - 1929	2.0	2.2	2.6	3.6	5.2	7.8	16.3	32.5	24.7	9.6	(5.7)	596.6
1929 - 1959	3.5	3.6	4.8	4.3	3.9	2.9	6.0	9.8	11.2	43.2	57.3	154.1
	Number of Transactions [f]											
1897 - 1914	117	95	67	53	32	22	12	12	8	7	7	1
1914 - 1929	112	93	59	40	28	19	10	6	6	6	6	1
1929 - 1959	274	228	144	113	86	70	40	26	20	8	6	1
	Average Transactions Per Year											
1897 - 1914	6.5	5.4	3.8	3.0	1.8	1.2	0.7	0.7	0.4	0.4	0.3	-
1914 - 1929	6.6	6.3	4.0	2.7	1.9	1.3	0.7	0.4	0.4	0.4	0.4	-
1929 - 1959	9.0	7.5	4.7	3.7	2.8	2.3	1.3	0.8	0.6	0.3	0.2	-
	Average Profit Per Year (%) [g] (Before Commissions)											
1897 - 1914	20.5	17.4	10.5	4.6	5.8	6.6	7.8	2.6	3.2	(3.3)	(3.9)	3.2
1914 - 1929	15.8	14.7	10.7	10.0	9.9	9.9	10.3	11.1	8.6	3.4	(2.1)	14.1
1929 - 1959	36.8	30.0	24.5	16.8	11.4	6.9	7.8	8.2	7.0	9.3	8.5	3.0

(Parentheses Signify Losses)

(a) Based on Dow Jones Industrials, 1897 - 1929 and Standard & Poor's Industrials, 1929 - 1959. See fn. (a), Table 5, and fn. (a), Table 6 for source references.

(b) Periods: January 2, 1897 to July 30, 1914
December 12, 1914 to September 3, 1929
September 7, 1929 to December 31, 1959.

(c) 5% filter here designates 5% in either direction, others designate indicated percentages upward and equal logarithmic moves downward. E.g., 10% filter implies 10% upward or 9.09% downward.

(d) Calculated as follows: for each move, as defined by a specified filter, let the variable, M, denote the difference between the logarithms of the upper and lower endpoints of the move. The figure given as the average move is 100 (antilog $\overline{M}$-1) where $\overline{M}$ is the arithmetic mean of the M's.

(e) Let $\overline{R}$ be the average logarithmic profit defined as $\overline{R} = \overline{M} - 2F$. $\overline{M}$ is defined in fn. (d) and F is log (1 + f) where f is the filter expressed as a ratio, e.g., f is 0.10 for a 10% filter. The quantity 2F corresponds to the portion of the move that is used up in getting in or out. On an upmove of average size $\overline{M}$ the percentage profit would be $P_u = 100(\text{antilog } \overline{R}-1)$; on a downmove, $P_d = 100\, P_u/(100 + P_u)$.
The average profit entered in Table 7 is $\overline{P} = (100 + P_u)^{\frac{1}{2}}(100 + P_d)^{\frac{1}{2}} - 100$.

(f) A transaction is defined as a purchase and sale, so that each transaction would require two commissions. In each period there is one terminal transaction, such as for Dec. 31, 1959, terminated not by a filter signal but by the period limits. The corresponding terminal move was counted as half a move in the computation of $\overline{M}$, and of the number of transactions per year.

(g) Computed as $100\left[1 + (\overline{P}/100)\right]^{\overline{q}} - 100$, where $\overline{q}$ is the average number of transactions per year, and $\overline{P}$ is defined in fn. (e).

length of time involved. In short, it substitutes the dimension of the "move" for the dimension of time.

The findings surveyed in this paper can be summarized by the statement that in speculative markets price changes appear to follow a random walk over time, but a move, once initiated, tends to persist. In particular, if the stock market has moved up x per cent it is likely to move up more than x per cent further before it moves down by x per cent. This proposition seems to be valid for x ranging from 5 per cent through 30 per cent. It will require further study to find out if it is valid for x smaller than 5%.

The riddle has been resolved. The statisticians' findings of a random walk over the time dimension is quite consistent with non-random trends in the move dimension. Such a trend does exist.

I leave to the speculation of others the question of what would happen to the effectiveness of the filter technique if everybody believed in it and operated accordingly.